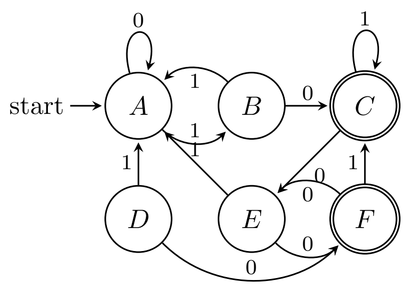
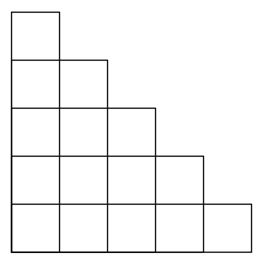
Name:

Sample Final Examination
MATH 562/CSCE 551
Spring Semester 2019

Problem 0 (Essential)
Consider the following DFA A over the alphabet $\{0, 1\}$:



(1) Give the table T of distinguishabilities for A. Show only the proper lower triangle of T, that is, fill in the following table with X in each entry corresponding to a pair of distinguishable



(2) Draw the minimal (4-state) DFA equivalent to A.

Problem 1
Consider the following two languages over $\{a, b\}$:

$$L_1 = \{w \mid w \text{ contains an odd number of } b\text{'s}\} ,$$

$$L_2 = \{w \mid \text{there exist two } b\text{'s in } w \text{ with an } a \text{ somewhere in between}\} .$$

(1) Give transition diagrams for DFAs recognizing L_1 and L_2 with 2 and 4 states, respectively.
(2) Using the product construction on your answer to (a), give an 8-state DFA recognizing $L_1 \cap L_2$. Do *not* perform any optimizations (e.g., removing unreachable states or transitions, or merging equivalent states).

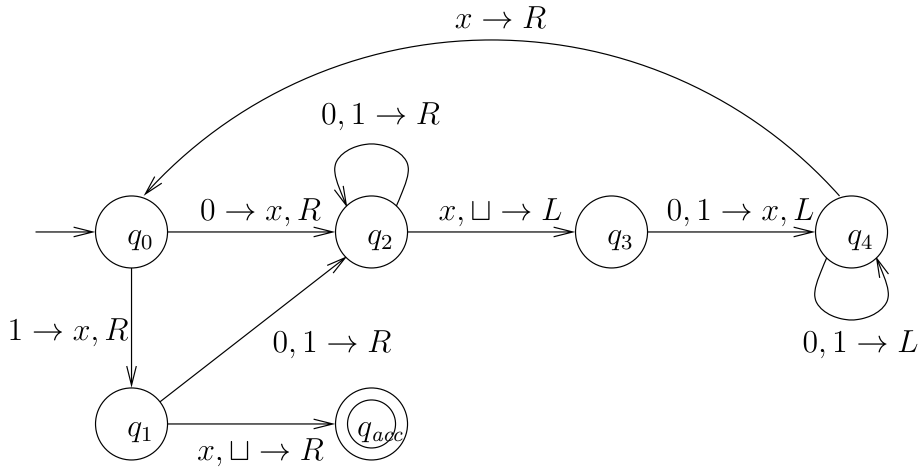
Problems 2 (Essential)
The alphabet in this problem contains the symbol 0. For any word w on our alphabet let NoZero(w) result from w by deleting all the 0's in w and then closing up the gaps. For any language L let NoZero(L) = $\{\text{NoZero}(w) \mid w \in L\}$. Prove that if L is a regular language, then so is NoZero(L).

Problem 3
Show that the language

$$L = \{w \in \Sigma^* \mid \text{no prefix of } w \text{ has more 0's than 1's}\}$$

is not pumpable (hence not regular). [Note: a string w is always a prefix of itself.]

Problem 4 (Essential)
Consider a standard, one-tape Turing machine M with input alphabet $\{0, 1\}$, tape alphabet $\{0, 1, x, \sqcup\}$ where $\sqcup$ is the blank symbol, and the following transition diagram:



Give the sequence of IDs (configurations) of the complete computation of M on each of the three inputs 011, 10, and 0. (Hint: M decides the language of all binary strings of odd length whose middle symbol is 1, so the first input is accepted while the other two are rejected.)

Problem 5
Fix some enumerator E that enumerates an infinite language. Let A be the language of all strings w so that

- E prints w at least once, and
- when w is first printed, it is longer than any string E has printed before.

Show that A is infinite and decidable. For the latter, give an explicit decision procedure for A.

Problem 6 (Essential)
Let f be a function that has the following property: for any TM M and string w such that M accepts w, $f(\langle M, w\rangle)$ outputs a number t such that M accepts w in less than t steps. (If M does not accept w, then $f(\langle M, w\rangle)$ could be any natural number.)

Show that no such f can be computable. [Hint: Show that if f were computable, then one can decide A_{TM}.]

Problem 7
Let f be a computable function. Show that range(f) is Turing-recognizable. (Here, range(f) is defined as $\{y \mid (\exists x)\ f(x) = y\}$).

Problem 8 (Essential)
Let $B := \{\langle M\rangle \mid M \text{ is a TM and } 010 \in L(M) \text{ and } 011 \notin L(M)\}$. Construct a mapping reduction from $\overline{A_{TM}}$ to B.

Problem 9
Let

$$L := \{w \mid \langle M, w\rangle \mid M \text{is a DFA that accepts } w^n \text{ for all natural numbers } n\}.$$

Show that $L \in \mathbf{P}$ by giving a polynomial time decision procedure for L.

Problem 10 (Essential)
The language

$$\text{VERTEXCOVER} := \{\langle G, k\rangle \mid G \text{ is a graph that has a vertex cover of size } k\}.$$

Show that VERTEXCOVER is NP-complete.

Problem 11
Prove that if TBFQ $\in \mathbf{NP}$, then $\mathbf{NP} = \mathbf{PSPACE}$.