

MATH 142 SECTION H02  
SPRING 2015

<b>Class:</b>	Tuesday and Thursday	11:40 a.m. to 12:55 p.m.	LeConte 121
	Monday	12:00 noon to 12:50 p.m.	Petigru 101
<b>Maple Lab:</b>	Wednesday	12:00 noon to 12:50	LeConte 401
<b>Instructor:</b>	George F. McNulty	Office: LeConte 302	777-7469 (office)
	781-9509 (Home)	e-mail: mcnulty@math.sc.edu	
<b>Office Hours:</b>	1:00 p.m. to 2:30 p.m.	Monday through Thursday	(Or by appointment)
	<b>Maple Lab Instructor:</b>	Chenfei Zhang	

**Text:** Calculus, Early Transcendentals, Sixth Edition  
**Author:** James Stewart

After reviewing some material in Chapter 5, we will cover most material in Chapters 6, 7, and 11, with a selection of material drawn from Chapters 8 and 10.

**Midterm Exams:**

Midterm I:	Thursday 29 January 2015	
Midterm II:	Thursday 19 February 2015	
Midterm III:	Thursday 19 March 2015	Please inform me as soon as possible
Midterm IV:	Thursday 16 April 2015	

**Final Exam:** Friday 3 May 2015  
9:00am–11:30am in LeConte 121

if these dates raise problems so that alternatives can be arranged.

WHAT YOU SHOULD LEARN IN THIS COURSE

The main outcome of your work in our course should be a mastery of integration of functions of one variable, both symbolically and through numerical approximation, and the acquisition of a conceptual understanding of limits and convergence for both functions and series. Your efforts should also give you a facility with integration and differentiation in polar coordinates. The attached sample final gives a detailed view of the skills and understanding you should have by the end of the course.

WHAT THE CLASS WILL BE DOING

Some of our time in class will be spent in discussion and working in small groups. For this reason, active personal participation is a key to the course. Your attendance and efforts will be needed during every meeting of the class.

Homework is at the heart of our course. Generally, an assignment will be due at the beginning of every lecture, recitation, and lab. Homework will be collaborative. The class will be divided into small teams for the purposes of homework.

Maple is a software package that is able to take over most of the drudge work associated with solving calculus problems. It is also able to provide insights into the underlying mathematical concepts through graphical and numerical methods. The Maple Lab associated with this course is a key resource for acquiring a mastery of the subject.

Every one of you is welcome to come to my office at anytime. I will generally be in every day, except Friday, from 9:30 am until 5 pm. On Friday I will be available between 11 a.m. and 3 p.m. While I have other responsibilities, your success is my first priority. Most of the time I will be able to set aside whatever I am doing, so don't hesitate to visit my office.

I hope you will find our course enjoyable, informative, and useful.

### **How Course Grades Will be Determined**

The objectives of this course can be broken down into 16 sorts of problems. Samples of these 16 sorts are attached below. The course Final will resemble this collection of sample problems. In turn these 16 sorts fall into two categories: core problems and those which lie outside the core. In addition to the problems on the examinations, the Maple Lab projects are also sorted into core projects and those that lie outside the core. Your grade for the course will be determined by how well you display mastery of these examination problems and Maple projects. For each sort of problem or project I identify three levels of performance: master level, journeyman level, and apprentice level. The examinations and labs given during the course provide you with opportunities to display your level of performance. The examinations will all be cumulative. The First Midterm will probably have 4 problems, the Second 8 problems (with 4 being variants of the ones occurring on the First Midterm), the Third Midterm will probably have 12 problems, and the Fourth Midterm as well as the Final will have 16.

I record how well you do on each problem and on each project (an M for master level, a J for journeyman level, an A for apprentice level) on each exam. After the Final, I make a record of the highest level of performance you have made on each sort of problem or project and use this record to determine your course grade. If you have at some point during the semester displayed a mastery of each of the 16 sorts of problems and on the Maple projects, then your grade will be an A. If you have at some point during the semester displayed a mastery of each of the core problems and projects, then you will get at least a C. The grade B can be earned by displaying mastery of all the core problems and mastery of about half of the rest of the problems and projects. The grade D will be assigned to anyone who can master several problems but has not yet displayed a mastery of all the core problems or projects. In borderline cases, the higher grade will be assigned to those students who turn in their homework regularly.

This particular way of arriving at the course grade is unusual. It has some advantages. Each of you will get several chances to display mastery of almost all the problems. Once you have displayed mastery of a problem there is no need to do problems like it on later exams. So it can certainly happen that if you do well on the midterms you might only have to do one or two problems on the Final. (It is not unusual that a few students do not even have to take the final.) On the other hand, because earlier weak performances are not averaged in, students who come into the Final on shaky ground can still manage to get a respectable grade for the course.

This method of grading also stresses working out the problems in a completely correct way, since accumulating a lot of journeyman level performances only results in a journeyman level performance. So it pays to do one problem carefully and correctly as opposed to trying get four problems partially correctly. Finally, this method of grading allows you to see easily which parts of the course you are doing well with, and which parts deserve more attention.

The primary disadvantage of this grading scheme is that it is complicated. At any time, if you are uncertain about how you are doing in the class I would be more than glad to clarify the matter.

SAMPLE FINAL EXAMINATION  
MATH 142  
FALL SEMESTER 2010  
INSTRUCTOR: PROF. GEORGE MCNULTY

PROBLEM 0.

In each part below find the indicated derivative or integral.

- a.  $D_x(\cos x^{\cos x})$
- b.  $D_x(2^{-x} - x^{-3})$
- c.  $\int \frac{(\ln x)^2}{x} dx$
- d.  $\int 5^{2x} dx$
- e.  $\int_0^{1/5} \frac{1}{1 + 25x^2} dx$

PROBLEM 1 (CORE).

The curves described by the equations

$$y = \frac{e^x + e^{-x}}{4}$$
$$y = \frac{e^x - e^{-x}}{2}$$
$$y = \frac{-e^x + e^{-x}}{2}$$

divide the plane into seven regions. Only one of these regions is bounded. Find its area.

PROBLEM 2 (CORE).

In each part below, find the volume of the solid described. Use calculus to justify your answers.

- a. The solid obtained by revolving an equilateral triangle of side  $s$  about one of its sides.
- b. A straight hole is bored through a very large wooden sphere. The drilling starts at the 'north pole' of the sphere, proceeds through the center, and emerges at the 'south pole' of the sphere. This hole is so wide that when its length is measured (from the rim at the north directly to the rim at the south) it turns out to be 6 inches. What remains, apart from the sawdust, is a ring-shaped solid. What is the volume of this solid?

**PROBLEM 3.**

In each part below, find the length of the curve described.

- a. The curve described by  $y = 4x^{3/2} + 1$  from  $x = 0$  to  $x = 9$ .
- b. The curve parameterized by:

$$\begin{aligned}x &= \sin t - \cos t \\y &= \sin t + \cos t \\ \pi/4 &\leq t \leq \pi/2\end{aligned}$$

**PROBLEM 4.**

Gasoline at a service station is stored in a cylindrical tank buried on its side, with the highest part of the tank 5 feet below the surface. The tank is 10 feet long and 6 feet in diameter. A cubic foot of gasoline weighs 45 pounds. Assume that the filler cap of each automobile gastank is 2 feet above the ground. How much work is done in emptying all the gasoline from the buried tank (which is initially full) into automobiles? [Hint:  $\int_{-3}^3 2\sqrt{9-y^2}dy$  gives the area of a circle of radius 3. This fact may prove useful at some point in your solution.]

**PROBLEM 5 (CORE).**

In each part below find the indicated antiderivative.

- a.  $\int x \cos 3x \, dx$
- b.  $\int \sin 2x \cos x \, dx$

**PROBLEM 6.**

In each part below find the indicated antiderivative.

- a.  $\int \frac{1}{x\sqrt{x^2-1}} \, dx$
- b.  $\int \frac{\sin^2 x}{\cos x} \, dx$

**PROBLEM 7 (CORE).**

In each part below find the indicated antiderivative.

- a.  $\int \frac{x^4 + x^3 + 5x^2 + 3x - 2}{x^3 - x} \, dx$
- b.  $\int \frac{3x^2 + 8x + 6}{x^3 + 3x^2 + 3x + 1} \, dx$
- c.  $\int \frac{x^2 + 2x + 1}{(x^2 + 4x + 5)^2} \, dx$

PROBLEM 8.

According to the definition of the natural logarithm we know  $\ln 4 = \int_1^4 \frac{1}{x} dx$ . Use this fact to do **one** of the parts below.

- Use the Trapezoid Rule with  $n = 4$  to produce an approximate value for  $\ln 4$ . Use the error term from the Trapezoid Rule to give a numerical bound on the error of your approximation.
- Use the Parabolic Rule (also known as Simpson's Rule) with  $n = 4$  to produce an approximate value for  $\ln 4$ . Use the error term from the Parabolic Rule to give a numerical bound on the error of your approximation.

PROBLEM 9.

In each part below, determine whether the integral exists, and if it does calculate its value. Please explain your reasoning.

- $\int_0^1 \frac{x}{\sqrt{1-x^2}} dx$
- $\int_0^\infty \frac{e^{-2\sqrt{x}}}{\sqrt{x}} dx$

PROBLEM 10 (CORE).

In each part below, determine whether the series converges. Please justify your answer.

- $\sum_{n=1}^{\infty} \frac{n}{n^2 + 4}$ .
- $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$ .
- $\sum_{n=1}^{\infty} \frac{n!2^n}{n^n}$ .
- $\sum_{n=1}^{\infty} \left(\frac{n}{n+1}\right)^n$ .

PROBLEM 11.

In each part below, determine whether the series converges absolutely, converges conditionally, or diverges. Please justify your answer.

- $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2 + 1}$ .
- $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^3 + 1}$ .
- $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ .

PROBLEM 12.

Let  $f(x) = \frac{1}{3x+2}$ .

- Find the Taylor polynomial of degree 5 for  $f(x)$  about  $a = 1$ .
- Find the remainder of the Taylor polynomial obtained in part (a) above.
- Find a numerical upper bound on the remainder found in part (b) above, under the stipulation that  $0 \leq x \leq 2$ .

PROBLEM 13 (CORE).

In each part below, find the interval of convergence of the given power series. Please show all your work.

- $\sum_{n=0}^{\infty} (-1)^n \frac{1}{n} x^n$ .
- $\sum_{n=1}^{\infty} \frac{n!}{(2n)!} x^n$ .
- $\sum_{n=0}^{\infty} n^n x^n$ .

PROBLEM 14.

In each part below, find a power series for the given function and determine its radius of convergence.

- $f(x) = \frac{1-x}{1+x}$ .
- $f(x) = (1+x) \tan^{-1} x$ .

PROBLEM 15.

Complete each part below

- Using polar coordinates, sketch the graph of

$$r = 2(1 - \cos \theta).$$

- Find the area of the region enclosed by the curve you sketched in part (a).