Homework: pp. 233-234 (6, 14, 18, 20)

In problem 6, find the area of the indicated inscribed or circumscribed polygon.

(6) See text for drawing

We know that the area of a rectangle is length times width. So, let's find the area of each rectangle and add them up to obtain a total area. To find the length (height) use the function to obtain an actual value.

**Area of Rectangle 1:**
\[
A = l \cdot w
\]
\[
A = 5(3\frac{1}{2}) \cdot \frac{1}{2}
\]
\[
= 9\frac{3}{8} \cdot \frac{1}{2}
\]

**Area of Rectangle 2:**
\[
A = l \cdot w
\]
\[
A = 5(4) \cdot \frac{1}{2}
\]
\[
= 3\frac{1}{2} \cdot \frac{1}{2}
\]

**Area of Rectangle 3:**
\[
A = l \cdot w
\]
\[
A = 5(3\frac{1}{2}) \cdot \frac{1}{2}
\]
\[
= 17\frac{1}{8} \cdot \frac{1}{2}
\]
Total Area:

\[ \frac{1}{2} \left( \frac{9 + 3 + 12 + 3}{8} \right) = \frac{31}{8} \]

In problem 14, find the area of the region under the curve \( y = f(x) \) over the interval \([a, b]\). To do this, divide the interval \([a, b]\) into \( n \) equal subintervals, calculate the area of the corresponding circumscribed polygon, and then let \( n \to \infty \).

14) \( y = x^2; \ a = -2, \ b = 2 \)

Consider the case \( a = 0 \) and \( b = 2 \),

\[ \Delta x = \frac{2}{n}, \quad x_i = \frac{2i}{n} \]

\[ f(x_i)\Delta x = \left( \frac{2i}{n} \right)^2 \left( \frac{1}{n} \right) = \frac{8i^2}{n^3} \]

\[ A(S_n) = \left[ \left( \frac{8}{n^3} \right) + \left( \frac{8(2^2)}{n^3} \right) + \cdots + \left( \frac{8(n^2)}{n^3} \right) \right] \]

\[ = \frac{8}{n^3} \left( 1^2 + 2^2 + \cdots + n^2 \right) = \frac{8}{n^3} \left[ \frac{n(n+1)(2n+1)}{6} \right] \]

\[ = \frac{4}{3} \left[ \frac{2n^3 + 3n^2 + n}{n^3} \right] \]

\[ = \frac{8}{3} + \frac{4}{n} + \frac{4}{3n^2} \]
\[
\lim_{n \to \infty} A(S_n) = \lim_{n \to \infty} \left( \frac{8}{3} + \frac{4}{n} + \frac{4}{2n^2} \right) = \frac{8}{3}
\]

By Symmetry, \( A = 2 \cdot \frac{8}{3} = \frac{16}{3} \).

18) Follow the directions of Problem 17 given that \( v = \frac{1}{3} t^2 + 2 \).

\[
S(t) \Delta t = \left[ \frac{1}{2} \left( \frac{t}{3} \right)^2 + 1 \right] \cdot \frac{1}{n} = \frac{1}{2n^3} + \frac{1}{n}
\]

\[
A(S_n) = \sum_{i=1}^{n} \left( \frac{1}{2n^3} + \frac{1}{n} \right) = \frac{1}{2n^3} \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n} \frac{1}{n}
\]

\[
= \frac{1}{2n^3} \left[ \frac{n(n+1)(2n+1)}{6} \right] + 1
\]

\[
= \frac{1}{12} \left[ 2 + \frac{3}{n} \cdot \frac{1}{n^2} \right] + 1
\]

\[
\lim_{n \to \infty} A(S_n) = \frac{1}{12} \cdot (2) + 1 = \frac{7}{6} \approx 1.17
\]

The object travel about 1.17 feet.

20) Suppose that an object, moving along the \( t \)-axis, has velocity \( v = t^2 \) meters per second at time \( t \) seconds. How far did it travel between \( t = 3 \) and \( t = 5 \)?
$A_3^5 = \frac{5^3}{3} - \frac{3^3}{3} = \frac{98}{3} \approx 32.7$

The object traveled about 32.7 meters.