In problem 10, a function $f$ and its domain are given. Determine the critical points, evaluate $f$ at these points, and find the (global) maximum and minimum values.

10) $f(x) = (x-1)^3 (x+2)^2; [-2, 2]$ 

Use the Product + Chain Rule! 

$f'(x) = 3(x-1)^2 (x+2)^2 + 2(x+2)(x-1)^3$

$3(x-1)^2 (x+2)^2 + 2(x+2)(x-1)^3 = 0$

$(x-1)^2 (x+2) [3(x+2) + 2(x-1)] = 0$

$(x-1)^2 (x+2) [3x+6 + 2x-2] = 0$

$(x-1)^2 (x+2) (5x+4) = 0$

$(x-1)^2 = 0$  
$x+2 = 0$  
$5x+4 = 0$

$x-1 = 0$  
$x = -2$  
$6x = -4$

$x = 1$  
$x = -2$  
$x = -4/5$

Critical points: $-2, -4/5, 1, 2$

$f(-2) = 0$
$f(-4/5) \approx -8.40$
$f(1) = 0$
$f(2) = 16$

Global minimum $f(-4/5) \approx -8.40$
Global maximum $f(2) = 16$
20. Find where the function \( g(t) = t^3 + \frac{1}{t} \) is increasing and where it is decreasing. Find the local extreme values of \( g \). Find the point of inflection. Sketch the graph.

\[ g(t) = t^3 + \frac{1}{t} \]

\[ g'(t) = 3t^2 - \frac{1}{t^2} \]

\[ 3t^2 = \frac{1}{t^2} \]
\[ 3t^4 = 1 \]
\[ t^4 = \frac{1}{3} \]
\[ t = \pm \frac{1}{3^{1/4}} \]

Critical points: \( \frac{1}{3^{1/4}} \) and \( 0 \) b/c \( g'(0) = \text{undefined} \).

\[ \begin{array}{ccc}
-1/3^{1/4} & 0 & 1/3^{1/4} \\
\end{array} \]

\[ g'(-2) = 11.75 \Rightarrow \text{increasing} \]
\[ g'(0.1) = -99.87 \Rightarrow \text{decreasing} \]
\[ g'(0.1) = -99.87 \Rightarrow \text{decreasing} \]
\[ g'(2) = 11.75 \Rightarrow \text{increasing} \]

Increasing Interval: \( (-\infty, -\frac{1}{3^{1/4}}) \) and \( (\frac{1}{3^{1/4}}, \infty) \)
Decreasing Interval: \( (-\frac{1}{3^{1/4}}, 0) \) and \( (0, \frac{1}{3^{1/4}}) \)

Local minimum \( g\left(-\frac{1}{3^{1/4}}\right) \approx -1.75 \)
Local maximum \( g\left(\frac{1}{3^{1/4}}\right) \approx 1.75 \)
$g(t)$ has no inflection point since $g(0)$ does not exist.

32) Sketch the graph of the given function $f$, in the region $(-\pi, \pi)$ unless otherwise indicated, labeling all extrema (local and global) and the inflection points and showing any asymptotes. Be sure to make use of $f'$ and $f''$.

$f(x) = \sin x - \tan x$

$f'(x) = \cos x - \sec^2 x$

$$\cos x - \sec^2 x = 0$$
$$\cos x = \sec^2 x$$
$$\cos x = \frac{1}{\cos^2 x}$$

$$\cos^3 x = 1 \quad x = 0 \quad \text{critical point}$$

$f''(x) = -\sin x - 2 \sec^3 x \tan x$

$$= -\sin x (1 + 2 \sec^3 x)$$

$f''(x) = 0 \text{ when } x = 0$
No local minima or maxima.
Inflection point at $(0,0)$

$f(x) = \sin x - \tan x = \sin x - \frac{\sin x}{\cos x}$

$$= \frac{\sin x \cos x - \sin x}{\cos x}$$

$\cos x = 0 \text{ when } x = -\frac{\pi}{2}, \frac{\pi}{2}$
Vertical Asymptotes occur at $x = -\frac{\pi}{2}$ and $x = \frac{\pi}{2}$