Homework: pp. 201-202 (6, 8, 16)

In problems 6, 18, and 16, a function is defined and a closed interval is given. Decide whether the Mean Value Theorem applies to the given function on the given interval. If it does, find all possible values; if not state the reason. In each problem, sketch the graph of the given function on the given interval.

6) \( F(x) = \frac{x^3}{3}; [-2, 2] \) \( F \) is continuous!

\[
F(2) = \frac{8}{3} \quad F(-2) = -\frac{8}{3}
\]

\[
\frac{F(2) - F(-2)}{2 - (-2)} = \frac{\frac{8}{3} + \frac{8}{3}}{4} = \frac{\frac{16}{3}}{4} = \frac{16}{3} \cdot \frac{1}{4} = 4 = F'(c)
\]

\[
F'(c) = \frac{3c^2}{3} = c^2
\]

\[
c^2 = \frac{16}{12} = \frac{4}{3}
\]

\[
c = \pm \frac{4}{\sqrt{3}} = \pm \frac{4}{2\sqrt{3}} = \pm \frac{2}{\sqrt{3}} \approx \pm 1.15
\]
8) \( F(t) = \frac{1}{t-1} \); \([0, 2]\)

The Mean Value Theorem does not apply
because \( F \) is not continuous on our
interval at \( t = 1 \).

16) \( C(\theta) = \csc \theta \); \([-\pi, \pi]\)

The Mean Value Theorem does not apply
because \( F \) is not continuous at \( \theta = -\pi, 0, \pi \).

Note: See webpage for all graph sketches.