14) \( \frac{d}{dx} \sqrt{\frac{x^2-1}{x^3-x}} \)

Let \( S(x) = \frac{\sqrt{x^2-1}}{\sqrt{x^3-x}} = \frac{\sqrt{x^2-1}}{\sqrt{x(x^2-1)}} = \frac{1}{\sqrt{x}} = x^{-1/2} \)

\( S'(x) = -\frac{1}{2} \cdot x^{-3/2} = -\frac{1}{2x^{3/2}} \)

20) \( \frac{d}{dt} [\sin^2(\cos 4t)] \)

Let \( S(t) = [\sin(\cos 4t)]^2 \)

\( S'(t) = 2[\sin(\cos 4t)] \cdot \cos(\cos 4t) \cdot (-\sin 4t) \cdot 4 \)

\( = -8[\sin(\cos 4t)] \cdot \cos(\cos 4t) \cdot \sin 4t \)

38) Find the coordinates of the point on the curve \( y = (x-3)^2 \) where there is a tangent line that is perpendicular to the line \( 2x-y+2=0 \).

First, we should find the slope of \( 2x-y+2=0 \). This is a linear equation, so convert it to \( y = mx+b \) form and find the slope.

\[ 2x - y + 2 = 0 \]

\[ y = -2 - 2x \]

\[ y = -(2+2x) \]

\[ y = 2x + 2 \quad m = -2 \]

To be perpendicular, the slope of the tangent line should be \(-\frac{1}{2}\) (negative reciprocal). So find the derivative which gives you the equation.
for the slope of the tangent line. Set that equal to \(-\frac{1}{2}\).

\[
y' = 2(x-2)
y' = 2x - 4
2x - 4 = -\frac{1}{2}
2x = -\frac{1}{2} + 4 = -\frac{1}{2} + \frac{8}{2} = \frac{7}{2}
\]

\[
\left(\frac{1}{2}\right) \cdot 2x = \frac{7}{2} \cdot \left(\frac{1}{2}\right)
\]

\[
x = \frac{7}{4}
\]

Find the y-coordinate from the equation of the curve to get your point.

\[
y = (x - 2)^2
\]

\[
y = \left(\frac{7}{4} - 2\right)^2 = \left(\frac{7}{4} - \frac{8}{4}\right)^2 = \left(-\frac{1}{4}\right)^2 = -\frac{1}{16} = \frac{1}{16}
\]

The coordinates of the point on the curve \(y = (x - 2)^2\) where there is a tangent line that is perpendicular to the line \(2x - y + 2 = 0, 15\) is \(\left(\frac{7}{4}, \frac{1}{16}\right)\).
42) An object is projected directly upward from the ground with an initial velocity of $128 \text{ ft/sec}$. Its height $s$ at the end of $t$ seconds is approximately $s = 128t - 16t^2$ feet.

a) When does it reach its maximum height and what is this height?

You must realize that if the object is at its maximum height, the velocity is 0. To get the velocity equation find $s'$.

$$s = 128t - 16t^2$$
$$s' = 128 - 32t$$

Set it equal to zero.

$$-32t + 128 = 0$$
$$-32t = -128$$
$$t = \frac{-128}{-32}$$

$t = 4$ seconds

At 4 seconds, the object is at its maximum height.

To find the maximum height, substitute 4 sec. into $t$ to find the height.

$$s = 128(4) - 16(16)$$
$$s = 256$$

The maximum height is 256 ft.

b) When does it hit the ground and with what velocity?

When an object is at ground level, the height is 0. Set $s = 0$. 
\[ s = 128t - 16t^2 \]
\[ 128t - 16t^2 = 0 \]
\[ -16t(t - 8) = 0 \quad \text{(Factor)} \]
\[ t - 8 = 0 \]
\[ t = 8 \]

The object hits the ground when \( t = 8 \) seconds.

To find the velocity at \( t = 8 \), find the velocity equation \( s'(t) \) and substitute \( 8 \) for \( t \).

\[ s = 128t - 16t^2 \]
\[ s'(t) = 128 - 32t \]
\[ s'(8) = 128 - 32(8) \]
\[ = -128 \text{ ft/sec} \]

When \( t = 8 \) seconds, the velocity is \(-128 \text{ ft/sec}\).