1. Compute the Laplacian eigenvalues of the following graph. (You can use Maple/Mathematica/Matlab etc.)

2. Find the Laplacian eigenvalues of the Cartesian product of two complete graphs \(K_s \square K_t\).

3. Show the Laplacian eigenvalues of \(P_n\) is \(1 - \cos \frac{k\pi}{n}\) for \(k = 0, 1, \ldots, n-1\).

4. A wheel graph \(W_n\) is a graph on \(n\) vertices, formed by connecting a single vertex to all vertices of an \((n-1)\)-cycle. Find the Laplacian eigenvalues of \(W_n\).

5. Suppose that a connected graph \(G\) has only two distinct eigenvalues. Prove that \(G\) is the complete graph.

6. (Bonus) Suppose \(G\) is a connected graph with \(\lambda_1 = 1\). Prove that \(G\) is a complete multi-partite graph.