Math 777 Homework 3

Due: March 14, before lecture.

1. Use the first moment method (i.e. using the expected value) to show

$$R(4, n) \geq c \left(\frac{n}{\log n} \right)^{3/2}$$

for some positive constant c.

2. Use the deletion method to show

$$R(4, n) \geq c \left(\frac{n}{\log n} \right)^{2}$$

for some positive constant c.

3. Use Lovász local lemma to show

$$R(4, n) \geq c \left(\frac{n}{\log n} \right)^{5/2}$$

for some positive constant c.

4. Let ϵ be a small positive constant and $p = n^{\epsilon - 1}$. Prove that almost surely the chromatic number of random graph $G(n, p)$ is at least

$$(1 - o(1)) \frac{np}{2 \ln(np)}.$$

5. Consider a random walk on the plane. At $t = 0$, a chip is at the origin. Each time a chip can move one step in a random chosen direction independently. Prove that with probability $1 - \frac{1}{n}$, the chip at time n is within the distance of $O(\sqrt{n \log n})$ from the origin.