## Math 777 Homework 2

Due: Feb. 9, before lecture.

1. Determine the crossing numbers of the Petersen graph.
2. Let $M_{2 n}$ be the graph obtained from the even cycle $C_{n}$ by adding chords joining vertices that are opposite. Determine the crossing number $\nu\left(M_{2 n}\right)$.
3. Decompose $K_{9}$ into three pairwise-isomorphic planar graphs.
4. Let $G$ be a matching of size $n$. Select a set of $k$ vertices at a random. Compute the expected number of edges induced by the selected vertices.
5. Prove that some 2-coloring of the edges of $K_{m, n}$ has at least $\binom{m}{r}\binom{n}{s} 2^{1-r s}$ monochromatic copies of $K_{r, s}$.
6. By examining common neighbors, prove that if $p>0$ is fixed and $o(n / \log n)$, then almost every $G(n, p)$ is $k$-connected.
