

Math 777 Homework 1

Due: Jan. 24, before lecture.

1. Prove that the complement of the 3-dimensional cube Q_3 is nonplanar.
2. Let G be the n -vertex simple planar graph with girth k . Prove G has at most $(n - 2)\frac{k}{k-2}$ edges. Use this to prove that the Petersen graph is nonplanar.

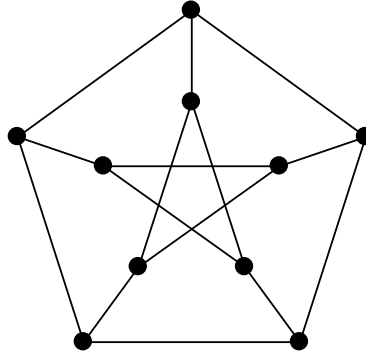


Figure 1: Petersen Graph

3. Determine the minimum number of edges that must be deleted from the Petersen Graph to obtain a planar subgraph.
4. A graph is *outerplanar* if it has an embedding with every vertex on the boundary of the unbounded face. Prove that an outerplanar graph with $n \geq 3$ vertices has at most $2n - 3$ edges.
5. Use the Four Color theorem to prove that every planar graph decomposes into two bipartite graphs.
6. Suppose that a simple planar graph G is k -regular and its dual graph G^* is also simple and l -regular. Find the solutions of all possible pairs of (k, l) and draw the plane graph G for every solution.