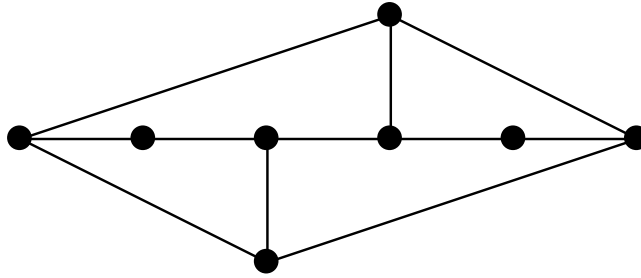


## Sample exam

The midterm exam on Oct. 28 is an open-book exam. You can bring books, notes, and homework. The exam covers everything other than extremal graph theory (i.e., lecture note 1-7). Exam problems should be easier than homework problems. The following problems serve a guideline of the exam.

1. Draw the tree on vertex set  $[8]$  with Prüfer code  $(473474)$ .
2. A graph  $G$  is called self-complementary if  $G \cong \bar{G}$ . Find two self-complementary graphs.
3. Calculate  $\alpha(G)$ ,  $\beta(G)$ ,  $\alpha'(G)$ ,  $\beta'(G)$  for the following graph  $G$ .



4. Suppose a connected graph  $G$  contains no subgraph isomorphic to  $P_4$ . Prove that there exists a vertex  $v$  such that  $N(v) = V(G) \setminus \{v\}$ .
5. An edge  $e$  of a graph  $G$  is  $\alpha$ -critical if  $\alpha(G - e) > \alpha(G)$ . Suppose that  $xy$  and  $xz$  are  $\alpha$ -critical edges in  $G$ . Prove that  $G$  has an induced subgraph that is an odd cycle containing  $xy$  and  $xz$ .