Homework 4

Due Monday February 11, 2008 at the beginning of class.

6. Let $R$ be the regular representation of $S_3$ over $\mathbb{C}$. Decompose $R$ into irreducible representations.

Remarks.
1. So, $R$ is the vector space

$$\mathbb{C}(1) \oplus \mathbb{C}(12) \oplus \mathbb{C}(13) \oplus \mathbb{C}(23) \oplus \mathbb{C}(123) \oplus \mathbb{C}(132),$$

and $\sigma$ in $S_3$ sends $\tau$ in $R$ to $\sigma \tau$ in $R$.

2. Sometime in the next few days we will establish Corollary 2.18 in Fulton-Harris. This corollary says that each irreducible representation $V_i$ of $S_3$ appears in $R$ exactly $\dim V_i$ times.

3. We already know all three irreducible representations of $S_3$. So we already know all of the numerology. I would like to see explicit irreducible submodules of $R$ that add up to $R$. 