MATH 702 – SPRING 2024
HOMEWORK 5

16. Let $K = \boldsymbol{k}(t)$ be the field of rational functions in one variable over the field $\boldsymbol{k}$. Let G be the subgroup of $\mathrm{Aut}_{\boldsymbol{k}} K$, which is generated by σ and τ where $\sigma(t) = \frac{1}{t}$ and $\tau(t) = 1 - \frac{1}{t}$. Find an element $g \in K$ with $K^G = \boldsymbol{k}(g)$. Prove your answer.

17. Let $\boldsymbol{k}$ be a field of characteristic p and let $f(x) = x^p - x - a \in \boldsymbol{k}[x]$. Suppose that $f(x) = 0$ has no solution in $\boldsymbol{k}$. Let K be a splitting field of $f(x)$ over $\boldsymbol{k}$. Is $\boldsymbol{k} \subseteq K$ a Galois extension? Find $\mathrm{Aut}_{\boldsymbol{k}} K$.

18. Give an example of a finite dimensional field extension $\boldsymbol{k} \subseteq K$ with an infinite number of intermediate fields. Also give an example of a finite dimensional field extension $\boldsymbol{k} \subseteq K$ with $K \neq \boldsymbol{k}[u]$ for any $u \in K$.