MATH 700
HOMEWORK 2

Due Friday, September 6, 1991 at the beginning of class

1. (Hoffman and Kunze, page 55, number 6.) Let $V$ be the vector space over the complex numbers of all functions from $\mathbb{R}$ to $\mathbb{C}$. Let $f_1(x) = 1$, $f_2(x) = e^{ix}$, and $f_3(x) = e^{-ix}$.
   (a) Prove that $f_1$, $f_2$, $f_3$ are linearly independent.
   (b) Let $g_1(x) = 1$, $g_2(x) = \cos x$, and $g_3(x) = \sin x$. Find an invertible matrix $P$ such that

\[
[f_1 \ f_2 \ f_3] P = [g_1 \ g_2 \ g_3].
\]

2. Let $V$ be a vector space of arbitrary dimension over the field $F$; let $B$ be a basis for $V$; and let $S$ be a linearly independent subset of $V$. Prove that there exists a subset $S_1$ of $B$ such that $S \cup S_1$ is a basis for $V$.

3. Let $V$ be the vector space of all polynomials in the variables $X_1, \ldots, X_n$ over the field $F$.
   a. What is the dimension of the subspace $W$ of $V$, which consists of all homogeneous polynomials of degree $d$?
   b. What is the dimension of the subspace $W'$ of $V$, which consists of all polynomials of degree at most $d$?