MATH 700
HOMEWORK 9

Due Friday, November 1, 1991 at the beginning of class.

1. Let $V$ be a vector space over the field $F$ and let $T: V \rightarrow V$ be a linear transformation. Suppose that $P(x)$ is a polynomial in $F[x]$ with $P(T)$ the zero transformation on $V$. Suppose further that $P(x) = Q_1(x)Q_2(x)$, where the ideal in $F[x]$ generated by $Q_1(x)$ and $Q_2(x)$ is equal to all of $F[x]$. Prove that $V = \ker Q_1(T) \oplus \ker Q_2(T)$.

2. Fill in the blank and prove the resulting statement. Let $V_1$ be a non-zero subspace of the vector space $V$. Every basis $B$ for $V$ has the property that $B \cap V_1 \neq \emptyset$ if and only if _____.

3. Fill in the blank and prove the resulting statement. Let $V_1$ and $V_2$ be subspaces of a vector space $V$. The set

$$S = \{v \in V_2 \mid v \notin V_1\} \cup \{0\}$$

is a subspace of $V$ if and only if _____.