1. (Hoffman and Kunze, page 115, number 4.) Let $V$ be a finite dimensional vector space over the field $F$ and let $T: V \rightarrow V$ be a linear transformation. Suppose that $c$ is an element of $F$ and $v$ is a nonzero vector in $V$ with $T(v) = cv$. Prove that there is a non-zero element $f \in V^*$ such that $T^*(f) = cf$.

2. Let $0 \xrightarrow{T_0} V_1 \xrightarrow{T_1} V_2 \xrightarrow{T_2} V_3 \xrightarrow{T_3} 0$ be an exact sequence of vector spaces over the field $F$. (That is, $\ker T_i = \operatorname{im} T_{i-1}$ for all $i$.) Prove that

$$0 \xrightarrow{T_3^*} \operatorname{Hom}_F(V_3, F) \xrightarrow{T_2^*} \operatorname{Hom}_F(V_2, F) \xrightarrow{T_1^*} \operatorname{Hom}_F(V_1, F) \xrightarrow{T_0^*} 0$$

is also an exact sequence of vector space.

3. Let $V$ be a vector space over an infinite field. Prove that $V$ is not equal to the union of a finite number of proper subspaces.