Quiz for June 25, 2007

Find a recurrence relation for the number of strings of zeros and ones of length \( n \) that contain a pair of consecutive zeros.

**ANSWER:** Let \( a_n \) equal the number of strings of zeros and ones of length \( n \) that contain a pair of consecutive zeros. Suppose \( n \geq 2 \). A string of zeros and ones of length \( n \) that contains a pair of consecutive zeros ends in either 1 (and the first \( n - 1 \) numbers contains a pair of consecutive zeros), or 10 (and the first \( n - 2 \) numbers contains a pair of consecutive zeros), or 00 (and the first \( n - 2 \) numbers can be anything). We conclude that \( a_n = a_{n-1} + a_{n-2} + 2^{n-2} \).