PRINT Your Name:___________________________________

Quiz for April 4, 2006

Solve the recurrence relation $a_n = 2a_{n-1} + 3^n$, with initial condition $a_1 = 5$. **Check your answer.**

ANSWER: We look for a solution of the homogeneous problem $a_n = 2a_{n-1}$ of the form $a_n = r^n$ for some r. We notice that $a_n = r^n$ is a solution of $a_n = 2a_{n-1}$ provided $r^n = 2r^{n-1}$, i.e., $r^n - 2r^{n-1} = 0$, i.e., $r^{n-1}(r-2) = 0$. So $r = 0$ (and that is not very interesting) or $r = 2$. We have found that $a_n = c2^n$ is the general solution of the homogeneous problem $a_n = 2a_{n-1}$. (Notice that the uninteresting solution $a_n = 0$ has indeed been included in our general solution.)

Now we look for a particular solution of the given non-homogeneous problem. We try for a solution of the form $a_n = A3^n$ for some constant A. Our candidate works if $A3^n = 2A3^{n-1} + 3^n$. Our candidate works if $3^{n-1}(A3 - 2A - 3) = 0$. Our candidate works if $A = 3$. At this point we know that the general solution of the original non-homogeneous linear recurrence relation is: $a_n = c2^n + 3^{n+1}$. We find c to make $a_1 = 5$: $5 = c2 + 9$; so, $c = -2$. Our answer is $a_n = -2(2^n) + 3^{n+1}$. In other words, our solution is $\boxed{a_n = 3^{n+1} - 2^{n+1}}$. We notice that $a_1 = 3^2 - 2^2 = 5$ ✓. Also,

$$2a_{n-1} + 3^n = 2(3^n - 2^n) + 3^n = 3(3^n) - 2(2^n) = 3^{n+1} - 2^{n+1} = a_n. \checkmark$$