Quiz for March 30, 2006

Find a recurrence relation for the number of strings of zeros and ones of length $n$ that contain a pair of consecutive zeros.

**Answer:** Let $a_n$ equal the number of strings of zeros and ones of length $n$ that contain a pair of consecutive zeros. Suppose $n \geq 2$. A string of zeros and ones of length $n$ that contains a pair of consecutive zeros ends in either 1 (and the first $n-1$ numbers contain a pair of consecutive zeros), or 10 (and the first $n-2$ numbers contain a pair of consecutive zeros), or 00 (and the first $n-2$ numbers can be anything). We conclude that $a_n = a_{n-1} + a_{n-2} + 2^{n-2}$. 