

9. (14 points) Write

$$\binom{600}{0} \binom{1100}{700} + \binom{600}{1} \binom{1100}{699} + \binom{600}{2} \binom{1100}{698} + \binom{600}{3} \binom{1100}{697} + \dots$$

$$\dots + \binom{600}{597} \binom{1100}{103} + \binom{600}{598} \binom{1100}{102} + \binom{600}{599} \binom{1100}{101} + \binom{600}{600} \binom{1100}{100}$$

as one binomial coefficient. Explain your work.

$$\binom{1700}{700}$$

Both numbers count the number of 700 member subcommittee which can be formed from a committee which consists of 600 men and 1100 women.

garbanut flies
plausible but wrong $\rightarrow 5$

10. (14 points) Solve the recurrence relation $a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 3$, with $a_0 = 0$, $a_1 = 3$, and $a_2 = 13$.

$$x^3 - 5x^2 + 8x - 4 = (x-1)(x-2)^2$$

so the gen. solⁿ is

$$a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 0 \Rightarrow$$

$$a_n = C_1 + C_2 2^n + C_3 n 2^n \quad \checkmark \quad 5$$

Look for a particular solution of the form A of the form

$$a_n = A n$$

$$A n - 5A(n-1) + 8A(n-2) - 4A(n-3) = 3$$

$$A(5 - 16 + 12) = 3$$

$$A(1) = 3$$

$$A = 3$$

so the general solution of $\textcircled{*}$ is

$$a_n = C_1 + C_2 2^n + C_3 n 2^n + 3n$$

$$0 = C_1 + C_2$$

$$3 = C_1 + 2C_2 + 2C_3 + 3$$

$$13 = C_1 + 4C_2 + 8C_3 + 6$$

$$\therefore C_1 = +7 \quad C_2 = -7 \quad C_3 = \frac{7}{2}$$

$$a_n = +7 - 7(2^n) + 7n 2^{n-1} + 3n$$

$$a_n = \dots \quad \underline{\underline{3}} \quad \uparrow \quad 7$$