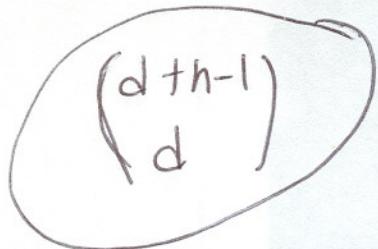


5. (14 points) How many monomials of degree d can be formed using the variables x_1, x_2, \dots, x_n ? (For example, the monomials of degree 3 in the variables x_1, x_2, x_3, x_4 are:

$$x_1^3, \quad x_2^3, \quad x_3^3, \quad x_4^3, \quad x_1^2 x_2, \quad x_1^2 x_3, \quad x_1^2 x_4, \quad x_2^2 x_1, \quad x_2^2 x_3, \quad x_2^2 x_4,$$

$$x_3^2 x_1, \quad x_3^2 x_2, \quad x_3^2 x_4, \quad x_4^2 x_1, \quad x_4^2 x_2, \quad x_4^2 x_3, \quad x_1 x_2 x_3, \quad x_1 x_3 x_4, \quad x_1 x_2 x_4, \quad x_2 x_3 x_4.)$$

This problem is the same as how many types of bags of candy can be made if we have n flavors and each bag gets d pieces.
(i.e. d picks $n-1$ switches)



6. (14 points) Prove that $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$.

We proceed by induction on n .

When $n=1$ the left side is 1. The right side is $\frac{1 \cdot 2 \cdot 3}{6} = 1$

Assume, by induction, that $\sum_{k=1}^{n-1} k^2 = \frac{(n-1)n(2n-1)}{6}$

We must show that $\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$. However,

$$\sum_{k=1}^n k^2 = \sum_{k=1}^{n-1} k^2 + n^2 = \frac{(n-1)n(2n-1)}{6} + n^2 = \frac{n}{6} \left[(n-1)(2n-1) + 6n \right]$$

$$= \frac{n}{6} \left[2n^2 - 3n + 1 + 6n \right] = \frac{n}{6} \left[2n^2 + 3n + 1 \right] = \frac{n}{6} [(n+1)(2n+1)] \quad \checkmark$$