(18) Let G be a graph, $\chi(G)$ be the chromatic number of G, and $\Delta(G) = \max\{\deg u \mid u \text{ is a vertex of } G\}$.

(a) Give an inequality ($\leq$) involving $\Delta(G)$ and $\chi(G)$ which holds for all graphs G.

$$\chi(G) \leq \Delta(G) + 1$$

(b) Give an example of a graph where $=$ holds in your formula from (a).

$\chi = 1$
$\Delta = 0$

(c) Give an example of a graph where $<$ holds in your formula from (a).

$\chi = 2$
$\Delta = 2$

(17) Let G be a graph, $\chi(G)$ be the chromatic number of G, and $\omega(G) = \max\{p \mid K_p \text{ is a subgraph of } G\}$.

(a) Give an inequality ($\leq$) involving $\omega(G)$ and $\chi(G)$ which holds for all graphs G.

$$\omega(G) \leq \chi(G)$$

(b) Give an example of a graph where $=$ holds in your formula from (a).

$\omega = 1$
$\chi = 1$

(c) Give an example of a graph where $<$ holds in your formula from (a).

$\omega = 2$
$\chi = 3$