

⑦ Let  $m$  and  $n$  be integers with  $m < n$  and let  $r$  be any number other than 1. Simplify the sum

$$\sum_{i=m}^n r^i = r^m + r^{m+1} + r^{m+2} + \dots + r^{n-2} + r^{n-1} + r^n$$

Let  $S = \text{sum}$

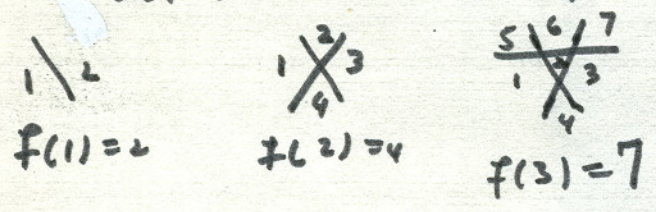
$$(1-r)S = r^m + r^{m+1} + \dots + r^{n-1} + r^n - r^{m+1} - \dots - r^{n-1} - r^n - r^{n+1}$$

$$= r^m - r^{n+1}$$

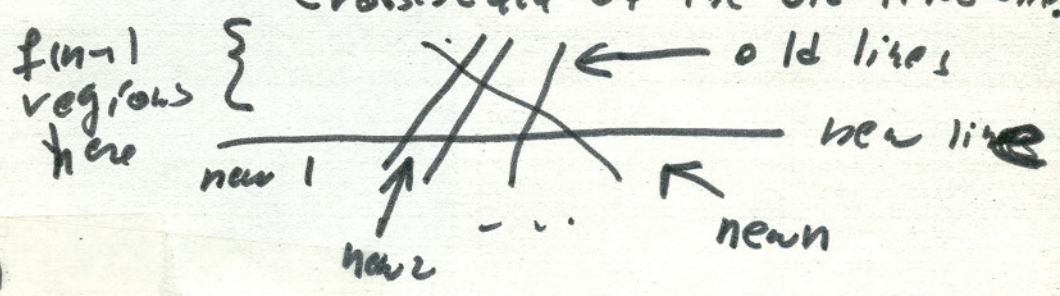
$$\therefore S = \frac{r^m - r^{n+1}}{1-r}$$

⑧ Suppose that  $n$  lines are drawn in the plane in "general position" (That is, no two lines are parallel and no three lines intersect at a point.) Into how many regions do these lines divide the plane? (Your final answer should be a function of  $n$ .)

Let  $f(n) = \#$  of regions when there are  $n$  lines



Suppose  $f(n-1)$  is known. Draw one more line. It crosses each of the old lines once.



$$f(n) = f(n-1) + n$$

$$f(n) = n + f(n-1) = n + (n-1) + f(n-2) = n + (n-1) + (n-2) + f(n-3)$$

$$= n + (n-1) + (n-2) + \dots + 2 + f(1) = (n + (n-1) + \dots + 1) + 1 =$$

$$\frac{n(n+1)}{2} + 1$$