

5. (17 points) Find a recurrence relation for the number of ways to arrange identical Toyotas, identical Hondas, identical Fords, and identical Cadillacs into n spaces if Toyotas and Hondas each take up one space, Fords take up two spaces, and Cadillacs take up three spaces. Do not forget to give sufficient initial conditions.

Let $a_n =$ the # of ways to arrange T, H, F, and C, as indicated into n spaces.

The collection of legal arrangements into n spaces may be partitioned as follows:
 last car is T or last car is H or last car is F or last car is C

Thus, $a_n = a_{n-1} + a_{n-1} + a_{n-2} + a_{n-3}$

So $a_n = 2a_{n-1} + a_{n-2} + a_{n-3}$
 $a_1 = 2$
 $a_2 = 5$
 $a_3 = 13$

Note The # of legal arrangements into n spaces with the last car as F is precisely a_{n-2} .

6. (17 points) What is the probability that a random arrangement of the letters in MISSISSIPPI has no consecutive vowels?

\mathcal{U} is all arrangements of M I I I I S S S S P P

$$\#(\mathcal{U}) = \frac{11!}{4! 4! 2!}$$

\mathcal{S} is all arrangements of M I I I I S S S S P P with no consecutive vowels

$$\#(\mathcal{S}) = \left(\begin{array}{l} \# \text{ of arrangements of } \\ 4 \text{ I's + 7 C's} \\ \text{with no consecutive I's} \end{array} \right) \cdot \left(\begin{array}{l} \# \text{ of ways to} \\ \text{bill in 7 c's} \\ \text{with M S S S S P P} \end{array} \right)$$

$$= \binom{8}{4} \frac{7!}{4! 2!}$$

$$\text{ans} = \frac{\binom{8}{4} \frac{7!}{4! 2!}}{11! / (4! 4! 2!)}$$

C _ C _ C _ C _ C _ C _ C _
 Pick 4 of the 8 _ to be I.
 The other 4 disappear