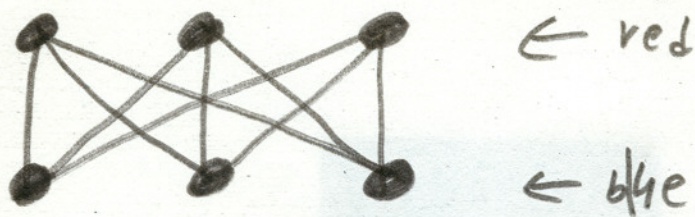


⑤ Give an example of a 2-colorable non-regular graph.

The utility graph



⑥ A graph has R components and e edges. If each component is a tree, how many vertices does the graph have? why?

Component i has n_i vertices e_i edges

component i is a tree so $n_i = e_i + 1$

The graph has e edges so $e_1 + \dots + e_R = e$

number of vertices = $n_1 + \dots + n_R = (e_1 + 1) + \dots + (e_R + 1) = e + R$

⑦ I am thinking of a connected graph G with 6 vertices.

(a) What is the fewest number of edges G can have? 5

(b) What is the most number of edges G can have? $1+2+3+4+5=15$

Justify your answers.

(a) Let T be a spanning Tree of G .

$n_T = n_G \quad e_T \leq e_G$

But $e_T + 1 = n_T = n_G = 6 \quad \therefore 5 = e_T \leq e_G$

(b) In K_6 every vertex is connected to every other vertex.

No Graph with 6 vertices can have more edges.

But K_6 has $5+4+3+2+1=15$ edges.