9 (a) Which binomial coefficient is eqsal to $\sum_{k=0}^{r} \binom{n+k}{k}$?

$$\binom{n+r+1}{r}$$

(b) Prove that your answer to (a) is correct.

Proof by induction on r;

If $r=0$ $\binom{n+0}{0}=1$ and $\binom{n+0+1}{0}=1$

If $r=1$ $\binom{n+0}{0}+\binom{n+1}{1}=n+2$ and $\binom{n+1+1}{1}=n+2$

Induction Hypothesis: suppose $\sum_{k=0}^{r}\binom{n+k}{k}=\binom{n+r+1}{r}$

We must show that $\sum_{k=0}^{r+1}\binom{n+k}{k}=\binom{n+(r+1)+1}{(r+1)}$

$$\text{L.H.S.} = \sum_{k=0}^{r}\binom{n+k}{k}+\binom{n+r+1}{r+1} \underset{\substack{\uparrow \\ \text{I.H.}}}{=} \binom{n+r+1}{r}+\binom{n+r+1}{r+1} = \binom{n+r+2}{r+1}$$

$\binom{n+r+2}{r+1}$ = R.H.S.

The trick from Pascal's triangle

$$\binom{n}{r-1} \quad \binom{n}{r}$$

$$\binom{n+1}{r}$$