Let $m, n,$ and $r$ be non-negative integers.

Which binomial coefficient is equal to \( \sum_{k=0}^{r} \binom{n}{k} \binom{m}{r-k} \)?

\[ \binom{n+m}{r} \]

Prove that your answer to (a) is correct.

**Combinatorial Proof:**

How many ways are there to pick a committee of $r$ people from $n + m$ people if $n$ of the people are non-managers and the other $m$ people are managers? We will answer the question correctly using two different approaches. Therefore, the two answers are equal.

**Approach 1:** There are \( \binom{n+m}{r} \) ways to pick an $r$-element subset of an $(n+m)$-element set.

**Approach 2** There are \( \binom{n}{0} \binom{m}{r} \) ways to pick a committee with

- $0$ non-managers and $r$ managers.
- $1$ non-manager and $r-1$ managers.
- $r$ non-managers and $0$ managers.

Altogether there are \( \sum_{k=0}^{r} \binom{n}{k} \binom{m}{r-k} \) ways to pick an $r$-element subset of an $(n+m)$-element set.