8 Let $m, n,$ and r be non-negative integers.

a which binomial coefficient is equal to $\sum_{k=0}^{r} \binom{n}{k}\binom{m}{r-k}$?

$$\binom{n+m}{r}$$

b <u>Prove</u> that your answer to a is correct.

<u>Combinatorial Proof</u>:

How many ways are there to pick a committee of r People from $n+m$ people if n of the People are non-managers and the other m people are managers? We will answer the question correctly using two different approaches. Therefore the two answers are **equal**.

<u>approach 1</u> There are $\binom{n+m}{r}$ ways to pick an r-element subset of an $(n+m)$-element set.

<u>approach 2</u> There are $\binom{n}{0}\binom{m}{r}$ ways to pick a committee with 0 non-managers and r managers.

There are $\binom{n}{1}\binom{m}{r-1}$ 1 $r-1$.

⋮

$\binom{n}{r}\binom{m}{0}$ r 0.

Altogether there are $\sum_{k=0}^{r} \binom{n}{k}\binom{m}{r-k}$ ways to pick an r-element subset of an $(n+m)$-element set.