however

$$a_n \underset{\substack{\uparrow \\ \text{The Definition} \\ \text{of } a_n}}{=} 2a_{n-1} + 4a_{n-2} \underset{\substack{\uparrow \\ IH}}{=} 2\left((1+\sqrt{5})^{n-1} + (1-\sqrt{5})^{n-1}\right) + 4\left((1+\sqrt{5})^{n-2} + (1-\sqrt{5})^{n-2}\right)$$

$$= (1+\sqrt{5})^{n-2}\left[4 + 2(1+\sqrt{5})\right] + (1-\sqrt{5})^{n-2}\left[4 + 2(1-\sqrt{5})\right]$$

Observe that $(1+\sqrt{5})^2 = 1 + 2\sqrt{5} + 5 = 4 + 2(1+\sqrt{5})$
and $(1-\sqrt{5})^2 = 1 - 2\sqrt{5} + 5 = 4 + 2(1-\sqrt{5})$

So $a_n = (1+\sqrt{5})^{n-2}(1+\sqrt{5})^2 + (1-\sqrt{5})^{n-2}(1-\sqrt{5})^2$

$$= (1+\sqrt{5})^{n+1} + (1-\sqrt{5})^{n+1} \checkmark$$

(3) (a) $n+m-1$

(b) Induct on n.

When $n=1$: It requires at most m comparisons in order to insert a_1 into $b_1 \le \dots \le b_m$. (At worst a_1 must be compared to every b_i). But $m = m+n-1$ when $n=1$.

Let $n \ge 1$ be fixed and $m \ge 1$ be arbitrary.

IH $\left[\begin{array}{l} \text{Assume, by induction, that at most } n+m-1 \\ \text{comparisons are needed when merging} \\ a_1 \le \dots \le a_n \text{ and } b_1 \le \dots \le b_m. \end{array}\right.$

We must prove that at most $(n+1)+m-1$ comparisons are needed to merge $a_1 \le \dots \le a_{n+1}$ and $b_1 \le \dots \le b_m$.