Math 574, 1992, Final Exam

PRINT Your Name:____________________

There are 11 problems on 6 pages. The exam is worth a total of 150 points.
SHOW your work. $\boxed{CIRCLE}$ your answers. CHECK your answers.

1. (13 points) How many integer solutions are there to the equality
$$x_1 + x_2 + x_3 + x_4 = 14,$$
with $1 \le x_1$, $2 \le x_2$, $0 \le x_3$, and $0 \le x_4$?

2. (13 points) A class contains 50 women and 20 men. Ten people are chosen at random. What is the probability that none are men?

3. (13 points) A True – False test consists of ten questions. If a student selects answers at random, then what is the probability that the student will guess at least 7 correct answers?

4. (13 points) Eight friends decide to have their picture taken. How many ways are there to arrange all eight people in a straight line, if John refuses to stand next to Mary?

5. (14 points) How many monomials of degree d can be formed using the variables $x_1, x_2, \ldots, x_n$? (For example, the monomials of degree 3 in the variables x_1, x_2, x_3, x_4 are:
$$x_1^3, \quad x_2^3, \quad x_3^3, \quad x_4^3, \quad x_1^2x_2, \quad x_1^2x_3, \quad x_1^2x_4, \quad x_2^2x_1, \quad x_2^2x_3, \quad x_2^2x_4,$$
$$x_3^2x_1, \quad x_3^2x_2, \quad x_3^2x_4, \quad x_4^2x_1, \quad x_4^2x_2, \quad x_4^2x_3, \quad x_1x_2x_3, \quad x_1x_3x_4, \quad x_1x_2x_4, \quad x_2x_3x_4.)$$

6. (14 points) Prove that $\displaystyle\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$.

7. (14 points) Prove that the sum of the fifth powers of any five consecutive nonnegative integers is divisible by 25.

8. (14 points) Write
$$\binom{300}{300} + \binom{301}{300} + \binom{302}{300} + \binom{303}{300} + \cdots + \binom{2997}{300} + \binom{2998}{300} + \binom{2999}{300} + \binom{3000}{300}$$
as one binomial coefficient. Explain your work.

9. (14 points) Write
$$\binom{600}{0}\binom{1100}{700} + \binom{600}{1}\binom{1100}{699} + \binom{600}{2}\binom{1100}{698} + \binom{600}{3}\binom{1100}{697} + \ldots$$
$$\cdots + \binom{600}{597}\binom{1100}{103} + \binom{600}{598}\binom{1100}{102} + \binom{600}{599}\binom{1100}{101} + \binom{600}{600}\binom{1100}{100}$$
as one binomial coefficient. Explain your work.

10. (14 points) Solve the recurrence relation $a_n - 5a_{n-1} + 8a_{n-2} - 4a_{n-3} = 3$, with $a_0 = 0$, $a_1 = 3$, and $a_2 = 13$.

11. (14 points) Find a CLOSED formula for $\displaystyle\sum_{k=0}^{n} k^4$.