

⑥ NO. Let $E_n = (-\frac{1}{n}, \frac{1}{n})$ for each $n \in \mathbb{N}$. Each E_n is an open set, but $\bigcap_{n=1}^{\infty} E_n = \{0\}$ and $\{0\}$ is not an open set.

⑦ Yes If $x \in \bigcup_{a \in A} E_a$, then $x \in E_{a_0}$ for some $a_0 \in A$.

But E_{a_0} is open so $\exists \epsilon N_\epsilon(x) \subseteq E_{a_0}$ Thus $N_\epsilon(x) \subseteq \bigcup_{a \in A} E_a$

Thus $\bigcup_{a \in A} E_a$ is open.