

There are 10 problems. Each problem is worth 10 points. SHOW your work.

CIRCLE your answer. **NO CALCULATORS!**

1. Show that $\mathbf{c}(t) = (\sin t, \cos t, e^t)$ is a flow line of the vector field $\vec{\mathbf{F}}(x, y, z) = (y, -x, z)$.
2. Find the divergence of the vector field $\vec{\mathbf{V}}(x, y, z) = x\vec{\mathbf{i}} + (y + \cos x)\vec{\mathbf{j}} + (z + e^{xy})\vec{\mathbf{k}}$.
3. Compute the curl of the vector field $\vec{\mathbf{F}}(x, y, z) = yz\vec{\mathbf{i}} + xz\vec{\mathbf{j}} + xy\vec{\mathbf{k}}$.
4. Find the equations of the line tangent to the curve traced out by $\mathbf{c}(t) = (t^3 + 1, e^{-t}, \cos(\frac{\pi t}{2}))$ at $t = 1$.
5. Express as an integral the arc length of the curve $x^2 = y^3 = z^5$ between $x = 1$ and $x = 4$ using a suitable parametrization. (Do not evaluate the integral.)
6. Find $\int_0^1 \int_1^{e^x} (x + y) dy dx$.
7. Find the volume of the region between $z = x^2 + y^2$ and $z = 50 - x^2 - y^2$.
8. Prove $\int_0^x \int_0^t F(u) du dt = \int_0^x (x - u)F(u) du$.
9. Find $\int_0^1 \int_x^1 e^{y^2} dy dx$.
10. Let D^* be the parallelogram, in the xy -plane, with vertices $(0, 0)$, $(2, -1)$, $(3, 2)$, and $(1, 3)$. Let D be the square

$$\{(u, v) \mid 0 \leq u \leq 1 \text{ and } 0 \leq v \leq 1\}.$$

Find a one-to-one function T from the xy -plane to the uv -plane such that D is the image of D^* under T .