Math 546, Exam 3, Summer 546

PRINT Your Name: __________________________

There are 10 problems on 5 pages. Each problem is worth 5 points.

I will put your exam outside my office door after I have graded it. You may pick it up any time before class on Monday. If I know your e-mail address, I will e-mail your score on Exam 3 to you.

1. Define “cyclic group”. Use complete sentences.

2. Define “generator of a group”. Use complete sentences.

3. State and prove Lagrange’s Theorem.

4. Prove that every subgroup of $\langle \mathbb{Z}, + \rangle$ is cyclic. I want a complete proof. “We did this in class” and “This follows from a Theorem we proved in class” are not good enough.

5. Let $(G, \ast)$ be an abelian group. Prove that the set

$$S = \{ g \in G \mid g \ast g = \text{id} \}$$

is a subgroup of $G$.

6. Let $G$ be the group $D_3$. (a) LIST the elements of the set

$$S = \{ g \in G \mid g \ast g = \text{id} \}.$$

(b) Is $S$ a subgroup of $G$? Justify your answer to (b).

7. Let $G$ be the group $U_2 \times U_4$. (a) LIST the elements of the set

$$S = \{ g \in G \mid g \ast g = \text{id} \}.$$

(b) Is $S$ a subgroup of $G$? Justify your answer to (b).

8. Give an example of an abelian, non-cyclic, group of order 16. I do not need to see many details.

9. Give an example of a non-abelian group of order 16. I do not need to see many details.

10. Find all of the subgroups of $\langle \mathbb{Z}_{12}, + \rangle$. Be sure to tell me why you know that you have all of the subgroups.