Exam 2, Math 546 Summer 2002

PRINT Your Name:_____

There are 10 problems on 5 pages. Each problem is worth 5 points.

I will put your exam outside my office door by noon on Friday. You may pick it up any time before class on Monday. If I know your e-mail address, I will e-mail your score on Exam 2 to you.

1. Define "group". Use complete sentences.

2. Define "center". Use complete sentences.

3. Define "centralizer". Use complete sentences.

4. Let A be the element $\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ in the group $G = \mathrm{GL}_2(\mathbb{R})$. Find the centralizer of A in G.

5. Let G be an abelian group. Prove that the set

$$S = \{g^3 \mid g \in G\}$$

is a subgroup of G.

6. Let G be the group D_3. (a) LIST the elements of the set

$$S = \{g^3 \mid g \in G\}.$$

(b) Is S a subgroup of G? Justify your answer to (b).

7. Let G be the group U_9. (a) LIST the elements of the set

$$S = \{g^3 \mid g \in G\}.$$

(b) Is S a subgroup of G? Justify your answer to (b).

8. Give an example of a group G and subgroups H and K of G with $H \cup K$ not equal to a subgroup of G. Explain.

9. Give an example of a group G and elements a and b of G with $(ab)^3 \neq a^3 b^3$. Explain.

10. The group D_4 has three distinct subgroups of order 4. List the elements of each of these subgroups. (I do not need to see any details.)