Math 546, Exam 4, Summer, 2001

PRINT Your Name:________________________________

There are 10 problems on 5 pages. Each problem is worth 5 points.

Neither your exam, nor your score, will not be available until class on Monday.

1. Are the groups $(\mathbb{Z}_{15}^{\times}, \times)$ and $(\mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2, +)$ isomorphic? Explain.

2. Are the groups $(\mathbb{Z}_2 \times \mathbb{Z}_3, +)$ and $(U_6, \times)$ isomorphic. Explain.

3. How many permutations in S_7 have order 6?

4. Write the cycle (12345) as a product of transpositions.

5. Consider $\varphi\colon \mathbb{Z}_9 \to \mathbb{Z}_3$, given by $\varphi([a]_9) = [a]_3$. Is φ a FUNCTION? Explain.

6. Suppose that $[a]_n = [a']_n$ and $[b]_n = [b']_n$. Prove $[ab]_n = [a'b']_n$.

7. Let $G = \mathbb{Z}_6 \times \mathbb{Z}_9$. LIST the elements of the set $\{g \in G \mid g + g + g = 0\}$. No explanation is needed.

8. True or False. If true prove it. If false, give a counterexample. If G is an abelian group and $H = \{x^3 \mid x \in G\}$, then H is a subgroup of G.

9. Let G be the group of non-zero complex numbers under multiplication. Let G' be the group of non-zero 2×2 matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, with real entries, under multiplication. Consider the function $\varphi\colon G \to G'$, which is given by $\varphi(a + bi) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$. Prove that φ is a group isomorphism.

10. Let $T = \mathbb{R} \setminus \{1\}$. Define $*$ on T by $a * b = ab - a - b + 2$. Prove that $(T, *)$ is isomorphic to $(\mathbb{R} \setminus \{0\}, \times)$.