Consider the operation $*$ on $\mathbb{Z}$ which is given by $a * b = \max\{a, b\}$. Is $(\mathbb{Z}, *)$ a group? Why or why not?

**ANSWER:**

$(\mathbb{Z}, *)$ is NOT a group. There is no identity element. We prove this statement by contradiction. Assume $id$, in $(\mathbb{Z}, *)$, is an identity element, then

$$id * (id - 1) = \max\{id, id - 1\} = id.$$ 

On the other hand, $id$ is the identity element of $(\mathbb{Z}, *)$, so $id * (id - 1) = id - 1$. Thus,

$$id = id - 1,$$

and $0 = -1$. This is impossible. Thus, $(\mathbb{Z}, *)$ does not have an identity element.