Let $G$ be a cyclic group and let $a$ and $b$ be elements of $G$ such that the equations $a = x^2$ and $b = x^2$ have no solution in $G$. Prove that $ab = x^2$ does have a solution in $G$.

**ANSWER:**

Let $g$ be a generator of $G$. The hypothesis $a = x^2$ has no solution in $G$ tells us that $a$ must equal $g^n$ for some odd integer $n$. In a similar manner, we see that $b = g^m$ for some odd integer $m$. We see that $ab = g^{n+m}$; furthermore, we know that $n + m$ is even. So, $n + m = 2p$ for some integer $p$; hence, $(g^p)^2 = ab$. 