Let $G$ be the group of rational numbers, under addition, and let $H$ and $K$ be subgroups of $G$. Assume that $H \neq \{0\}$ and $K \neq \{0\}$. Prove that $H \cap K \neq \{0\}$.

**ANSWER:** The hypothesis ensures that there is a number $\frac{a}{b}$ in $H$ with $a$ and $b$ positive integers. In a similar way we see that there is a number $\frac{c}{d}$ in $K$, with $c$ and $d$ positive integers. Observe that $ac$ is a positive integer in $H \cap K$. Indeed, the group $H$ is closed and $\frac{a}{b} \in H$. Add $\frac{a}{b}$ to itself $bc$ times to see that $ac$ is also in $H$. The group $K$ is closed and $\frac{c}{d} \in K$. Add $\frac{c}{d}$ to itself $ad$ times to see that $ac$ is in $K$. We conclude that $ac$ is in $H \cap K$. 