section 3.1, page 90
1, 2, 3, 6, 7, 11, 14, 15, 20 (I ignore the hint.), 21.

section 3.2, page 101
4a, 10, 11, 12 (Do 12 for two subgroups. Also, suppose $H$ and $K$ are subgroups of the group $G$. Is the union $H \cup K$ a subgroup of $G$? If so, prove the statement. If not, give an example.), 13, 14ab (In (c) and (d), replace $S_3$ with $D_4$. Find $C(\rho)$. Find $C(\rho^2)$. Find $C(\sigma)$.), 15, 16 (replace $S_3$ with $D_3$ and $D_4$), 18 (What happens if the hypothesis that $G$ is a cyclic group is removed. Is the statement still true? If so, prove it. If not, find a counter example.), 21, 22.

1. Find 6 subgroups of $D_4$ in addition to $D_4$ and \{id\}.

2. Let $\rho$ be rotation counter clockwise by $120^\circ$ fixing the origin. Let $\sigma$ be reflection of the $xy$ plane across the $x$ axis. Let $D_3$ be the smallest subgroup of the group of rigid motions which contains $\rho$ and $\sigma$.
   (a) List the elements of $D_3$.
   (b) Find the multiplication table for $D_3$.
   (c) Describe the action of each element of $D_3$.
   (d) Show that if $\tau \in D_3$, then $\tau(T) = T$, where $T$ is the triangle with vertices $(1,0)$, $(-\frac{1}{2}, \frac{\sqrt{3}}{2})$, and $(-\frac{1}{2}, -\frac{\sqrt{3}}{2})$.

3. Find three subgroups of $D_4$ of order 4.

4. Let $U_8$ be the group of complex numbers which satisfy $x^8 = 1$. Find two subgroups of $U_8$ in addition to \{id\} and $U_8$.

5. Let $G$ be the group $U_9$, which consists of all complex numbers $z$ such that $z^9 = 1$.
   (a) What is the order of each element of $G$?
   (b) Which elements of $G$ are generators of all of $G$. (Recall that the element $g$ in the group $G$ generates $G$, if $\langle g \rangle = G$.)
   (c) Which elements $g$ of $G$ can be written in the form $h^2$ for some $h \in G$?
   (d) Which elements $g$ of $G$ can be written in the form $h^3$ for some $h \in G$?

6. Let $H$ be a subgroup of the group $G$. We will prove Lagrange’s Theorem by introducing an equivalence relation “$\sim$” on $G$. We will say that if $x$ and $y$ are in $G$, then $x \sim y$ if and only if $xy^{-1} \in H$. We will prove that $G$ is equal to the union of disjoint subsets where every element in a given subset is equivalent to every other element in the subset. One says that we have decomposed $G$ into a disjoint union of equivalence classes.
(a) Let $G = U_9$ and let $H$ be the subgroup of $G$ which is generated by $u^3$, where $u = \cos \frac{2\pi}{9} + i \sin \frac{2\pi}{9}$. Decompose $G$ into a disjoint union of equivalence classes, as described above, for this $G$ and $H$.

(b) Let $G = D_4$ and let $H$ be the subgroup of $G$ which is generated by $\sigma$. Decompose $G$ into a disjoint union of equivalence classes, as described above, for this $G$ and $H$. 

section 1.4, page 40 1b, 2a, 3b.

section 3.1, page 90 4, 5.

section 3.2, page 101 3bc, 8.

section 2.3, page 75 1a, 3 (for $\sigma \tau$), 4, 5a, 6, 7.


section 3.3, page 110 4, 5, 6, 7.

section 3.4, page 118 4, 5, 6, 7, 10, 12, 13, 14, 16, 17.

section 3.5, page 125 4, 6, 11.

section 3.7, page 145 3, 6, 12.

section 3.8, page 155 4, 12, 18, 19, 22.