1. DEFINE group.

2. DEFINE cyclic group.

3. DEFINE the center of a group.

4. DEFINE normal subgroup.

5. STATE Lagrange’s Theorem.

6. STATE the lemma from number theory about linear combinations and greatest common divisors.

7. STATE the “Chinese Remainder Theorem” about the group $\mathbb{Z}_n \times \mathbb{Z}_m$ and the group $\mathbb{Z}_{nm}$.

8. STATE the lemma about the order of the element $ab$ in terms of the order of $a$ and the order of $b$.

9. STATE the two results about the subgroups of a cyclic group.

10. Pick one of the statements from problems 5 through 9. Tell me which statement you have chosen. PROVE the statement.

11. Pick a second statement from problems 5 through 9. Tell me which statement you have chosen. PROVE the statement.

12. What is the order of the element $(\cos \frac{\pi}{3} + i \sin \frac{\pi}{3}, \rho)$ in the group $U_6 \times D_4$? Explain your answer.

13. Let $G$ be the group $\mathbb{Z}_4 \times \mathbb{Z}_{10}$. Let $N$ be the subgroup $<(2, 2)>$ of $G$. What is the order of the element $(1, 2) + N$ in the group $\frac{G}{N}$? Explain your answer.

14. Let $(\mathbb{R}^+, \times)$ represent the group of positive real numbers under multiplication. Does $(\mathbb{R}^+, \times)$ contain any non-cyclic subgroups? If not, explain why not. If so, exhibit such a subgroup and explain why the subgroup is not cyclic.

15. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) All groups of order 6 are isomorphic.
16. TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.)
All groups of order 7 are isomorphic.

17. How many permutations in $S_6$ have order 4. Explain your answer.

18. Let $G$ be the group of non-zero complex numbers under multiplication. Let $G'$ be the group of non-zero $2 \times 2$ matrices of the form $\begin{bmatrix} a & b \\ -b & a \end{bmatrix}$, with real entries, under multiplication. Consider the function $\varphi: G \to G'$, which is given by $\varphi(a + bi) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$. Prove that $\varphi$ is a group isomorphism.

19. Let $K$ and $N$ be subgroups of the group $G$. Let $S = \{kn \mid k \in K \text{ and } n \in N\}$.

If $N$ is a normal subgroup of $G$, then prove that $S$ is a subgroup of $G$.

20. The subgroup $N = \{\text{id}, (12)(34), (13)(24), (14)(23)\}$ of the group $S_4$ is normal. The factor group $\frac{S_4}{N}$ is isomorphic to which familiar group? Explain your answer.