Math 546, Exam 4, Spring, 2001

PRINT Your Name:________________________________

There are 10 problems on 5 pages. Each problem is worth 10 points.

1. Define *group isomorphism*.

2. Let H be a non-zero subgroup of $(\mathbb{Z},+)$. Prove that H is a cyclic group. (I want you to write down a complete proof. "We did this in class" is not a satisfactory answer.)

3. Let m and n be positive integers. Let H be the set of all linear combinations $an+bm$, where a and b are integers. It can be shown that there exists a positive element $h \in H$, so that every element of H is a multiple of h. PROVE that h is the greatest common divisor of m and n. (I am not asking you to prove the existence of h. I am saying, "Suppose h exists. Now prove that h is the g.c.d.".)

4. Give an example of a group G and elements a and b in G of finite order with the order of ab not equal to the order of a times the order of b.

5. Let a and b be elements of finite order in a group G. LIST two hypothesis so that if a and b satisfy these hypotheses, then the order of ab is equal to the order of a times the order of b. PROVE the result.

6. How many permutations in S_7 have order 3? Explain your answer.

7. Let $\mathbb{Z}$ be the group of integers under addition and let H be the subgroup of even integers. Are the groups $\mathbb{Z}$ and H isomorphic? Explain your answer.

8. Are the groups $\mathbb{Z}_{15}$ and $\mathbb{Z}_3 \times \mathbb{Z}_5$ isomorphic? (The operation in each of the groups $\mathbb{Z}_{15}$, $\mathbb{Z}_3$, and $\mathbb{Z}_5$ is addition.) Explain your answer.

9. Are the groups $\mathbb{Z}_8$ and $\mathbb{Z}_2 \times \mathbb{Z}_4$ isomorphic? (The operation in each of the groups $\mathbb{Z}_8$, $\mathbb{Z}_2$, and $\mathbb{Z}_4$ is addition.) Explain your answer.

10. Which of the groups $(\mathbb{C} \setminus \{0\}, \times)$, $(\mathbb{R}, +)$, $(\mathbb{R} \setminus \{0\}, \times)$, and $(\mathbb{R}^{\text{pos}}, \times)$ are isomorphic? (Be sure to examine each pair of groups. I use $\mathbb{R}^{\text{pos}}$ to represent the set of positive real numbers.) Explain your answer.