Math 546, Exam 2, Spring, 2001

PRINT Your Name:________________________________

There are 7 problems (10 parts) on 5 pages. Each **part** is worth 10 points.

1. State Lagrange's Theorem.

2. Define "cyclic group".

3. Let G be a group and $a \in G$.
   (a) Define "the centralizer of a".
   (b) Prove that the centralizer of a is a subgroup of G.
   (c) Let $G = D_4$ and $a = \rho$. Find the centralizer of a.

4.
   (a) Give an example of a cyclic subgroup of D_4 of order 4. No proof is necessary.
   (b) Give an example of a subgroup of D_4 of order 4 which is not cyclic. No proof is necessary.

5. Let $U_8 = \{z \in \mathbb{C} \mid z^8 = 1\}$. Which elements in U_8 have the form x^3 for some $x \in U_8$? Explain your answer.

6. Let $G = D_4$ and $H = \{\text{id}, \sigma\rho^3\}$. Find 4 elements x_1, x_2, x_3, x_4 of G so that G is the disjoint union $[x_1] \cup [x_2] \cup [x_3] \cup [x_4]$, where $[x] = \{y \in G \mid xy^{-1} \in H\}$. Identify x_1, x_2, x_3, x_4 and show the full set $[x_i]$ for each i.

7. True or False (If true, then prove it. If false, then give a counterexample.) If every proper subgroup of the group G is abelian, then G is abelian. (Recall that the subgroup H is G is a *proper* subgroup if $H \neq G$.)