1. Let $G$ be a group with at least two elements. Suppose that the only subgroups of $G$ are $G$ and $\{e\}$. PROVE that $G$ is a finite cyclic group of prime order.

2. Let $H$ be a subgroup of the group $G$. Suppose that $a$ and $b$ are elements of $G$ with $aH = bH$. Does $a^2H$ HAVE TO EQUAL $b^2H$. If your answer is “yes”, then PROVE the statement. If your answer is “no”, then give a counterexample.

3. Let $\varphi: G \to G'$ and $\gamma: G' \to G''$ be group homomorphisms. Prove that the composition $\gamma \circ \varphi$ is a group homomorphism from $G$ to $G''$.

4. Let $x$ be a fixed element of the group $G$. Define the function $\varphi: G \to G$ by $\varphi(g) = xgx^{-1}$ for all $g$ in $G$. Prove that $\varphi$ is a group homomorphism.

5. Let $G$ be the direct product $U_3 \times U_5$. Is $G$ a cyclic group? WHY? or WHY NOT?