Math 546, Exam 3, Summer, 1993

Use your own paper. Each problem is worth 10 points.

1. Let G be a group with at least two elements. Suppose that the only subgroups of G are G and $\{e\}$. PROVE that G is a finite cyclic group of prime order.

2. Let H be a subgroup of the group G. Suppose that a and b are elements of G with $aH = bH$. Does a^2H HAVE TO EQUAL b^2H. If your answer is "yes", then PROVE the statement. If your answer is "no", then give a counterexample.

3. Let $\varphi\colon G \to G'$ and $\gamma\colon G' \to G''$ be group homomorphisms. Prove that the composition $\gamma \circ \varphi$ is a group homomorphism from G to G''.

4. Let x be a fixed element of the group G. Define the function $\varphi\colon G \to G$ by $\varphi(g) = xgx^{-1}$ for all g in G. Prove that φ is a group homomorphism.

5. Let G be the direct product $U_3 \times U_5$. Is G a cyclic group? WHY? or WHY NOT?