Math 546, Exam 2, Summer, 1993

Use your own paper. Each problem is worth 10 points.

1. Let $\sigma = (1,2,3)(4,5,6)$ and $\tau = (3,4,5)$ be elements of S_6. Write $\tau\sigma\tau^{-1}$ as the product of disjoint cycles.

2. Let H be a subgroup of S_n for some $n \geq 2$. Prove that either every permutation in H is even or exactly half of the permutations in H are even.

3. Let H be a subgroup of the group G. Let a be a fixed element of G and let

$$K = \{aha^{-1} \mid h \in H\}.$$

   Prove that K is a subgroup of G.

4. Let A be a set, B be a subset of A, and b be an element of B. Is

$$\{\sigma \in S_A \mid \sigma(b) \in B\}$$

   always a subgroup of S_A? If your answer is yes, then PROVE the statement. If your answer is no, then give a COUNTEREXAMPLE.

5. Let A be a set and b be an element of A. Is

$$\{\sigma \in S_A \mid \sigma(b) = b\}$$

   always a subgroup of S_A? If your answer is yes, then PROVE the statement. If your answer is no, then give a COUNTEREXAMPLE.