5. Let \((G, \ast)\) be an abelian group. Prove that the set 
\[ S = \{ g \in G \mid g \ast g = \text{id} \} \]
is a subgroup of \(G\).

Close
Take \(x\) and \(y \in S\). We know \(x \ast x = \text{id}\) and \(y \ast y = \text{id}\). We now show
\(x \ast y \in S\). Observe that 
\[(x \ast y) \ast (x \ast y) = (x \ast x) \ast (y \ast y) = \text{id}\]
because \(G\) is abelian.

\(x \ast y \in S\) because \(x \ast x = \text{id}\)

Inverses: take \(x \in S\) so \(x \ast x = \text{id}\) unitality on both sides by \(x^{-1} \ast x^{-1}\)
to get \(\text{id} = x^{-1} \ast x^{-1}\).

Thus \(S\) is a subgroup of \(G\).

6. Let \(G\) be the group \(D_3\). (a) LIST the elements of the set
\[ S = \{ g \in G \mid g \ast g = \text{id} \}. \]

(b) Is \(S\) a subgroup of \(G\)? Justify your answer to (b).

\(\text{NO. } S\) is not closed, since \(\sigma\) \(\ast \sigma = \text{id}\) but \(\sigma \not\in S\).