5. Let G be an abelian group. Prove that the set

$$S = \{g^3 \mid g \in G\}$$

is a subgroup of G.

Closure Take x and y from S so $x = g^3$ and $y = h^3$ for some g and h in G. We see that $xy = g^3h^3 = (gh)^3$ because G is abelian. Thus xy is a perfect cube and therefore $xy \in S$.

id $id = id^3$ so $id \in S$

inverses Take $x \in S$. So $x = g^3$ for some $g \in G$. Well g has an inverse, call it g^{-1}. We know that $(g^{-1})^3$ is x's inverse. Thus x inverse is a perfect cube and $x^{-1} \in S$.

6. Let G be the group D_3. (a) LIST the elements of the set

$$S = \{g^3 \mid g \in G\}.$$

(b) Is S a subgroup of G? Justify your answer to (b).

$id^3 = id$
$\sigma^3 = \sigma$
$(\sigma\rho)^3 = \sigma\rho$
$(\sigma\rho^2)^3 = \sigma\rho^2$
$\rho^3 = id$
$(\rho^2)^3 = id$

$S = \{id, \sigma, \sigma\rho, \sigma\rho^2\}$

I see that S is not closed because $\sigma \in S$ $\sigma\rho \in S$ but $\rho = \sigma(\sigma\rho)$ is not in S

Thus S is not a subgroup of D_3