82

19. Let K and N be subgroups of the group G. Let

$$S = \{kn \mid k \in K \text{ and } n \in N\}.$$

If N is a normal subgroup of G, then prove that S is a subgroup of G.

closure $kn k'n' = kk' \underbrace{(k')^{-1} n k'}_{\text{in } N \text{ because } N \triangleleft G} n' \in S$

inverses kn is a typical element of S. The inverse of kn is

$n^{-1}k^{-1} = k^{-1} \underbrace{k n^{-1} k^{-1}}_{\text{in } N \text{ because } N \triangleleft G} \in S$

id $\text{id} \cdot \text{id} \in S$

20. Let G be an abelian group. Let $H = \{x^2 \mid x \in G\}$. Prove H is a subgroup of G.

id $\text{id} \in G$ so $\text{id} = \text{id}^2 \in H$

closure Take $x^2, y^2 \in H$ we see $x^2 y^2 \underset{G \text{ is abelian}}{=} (xy)^2 \in H$

inverses Take $x^2 \in H$. The inverse of x^2 is $(x^{-1})^2$, which is in H.