15. How many permutations in $S_8$ have order 4. Explain your answer.

4-cycles have order 4. A four cycle times a disjoint 2-cycle has order 4.
No other elements of $S_8$ have order 4.

There are $\binom{6}{4} \cdot 3!$ 4-cycles in $S_6$.

Ways to choose 4 numbers out of 6 numbers

$= \binom{6}{4} \cdot \binom{2}{2}$

$= \binom{6}{4} \cdot 1$

There are $2 \cdot \binom{6}{4} = 2 \cdot 5 \cdot 3 = 30$ elements of order 4 in $S_6$.

16. What is the inverse of $[39]_{83}$ in $(\mathbb{Z}_{83}^\times, \times)$. Check your answer.

$83 = 2 \cdot 39 + 5$
$39 = 7 \cdot 5 + 4$
$5 = 1 \cdot 4 + 1$

So $1 = 8 \cdot 1 - 2 \cdot 5$
$1 = 8 \cdot 1 - 1 \cdot 39$
$1 = 8 (83 - 2 \cdot 39) - 1 \cdot 39$
$1 = 8 \cdot 83 - 17 \cdot 39$

$8 \cdot 83 = 664$
$39 \div 17$
$273 \div 39$
$66 \cdot 3$

$[-17]_{83}$ is the inverse of $[39]_{83}$ in $\mathbb{Z}_{83}^\times$. 

$[39]_{83} \cdot [-17]_{83} = 1$.