

15. How many permutations in S_6 have order 4. Explain your answer.

4-cycles have order 4. A four cycle times a disjoint 2-cycle has order 4.
No other elements of S_6 have order 4.

There are $\binom{6}{4} \cdot 3!$ 4-cycles in S_6

ways to choose
4 numbers out of
6 numbers

(1 smallest, 2 choices, 2 choices, 1)

There are $\binom{6}{4} \cdot 3!$ (---)(---) in S_6

Put smallest
remaining
here

There are $2 \cdot \binom{6}{4} \cdot 3! = 2 \cdot 15 \cdot 6 = 180$ elements of order 4 in S_6

16. What is the inverse of $[39]_{83}$ in $(\mathbb{Z}_{83}^\times, \times)$. Check your answer.

$$\begin{aligned} 83 &= 2 \cdot 39 + 5 \\ 39 &= 7 \cdot 5 + 4 \\ 5 &= 1 \cdot 4 + 1 \end{aligned}$$

$[-17]_{83}$ is the inverse of $[39]_{83}$ in \mathbb{Z}_{83}^\times

$$\begin{aligned} \text{so } 1 &= 1 \cdot 5 - 1 \cdot 4 \\ 1 &= 1 \cdot 5 - 1(39 - 7 \cdot 5) \\ 1 &= 8 \cdot 5 - 1 \cdot 39 \\ 1 &= 8(83 - 2 \cdot 39) - 1 \cdot 39 \\ 1 &= 8 \cdot 83 - 17 \cdot 39 \end{aligned}$$

$$\text{or } 8 \cdot 83 = 664$$

$$\begin{array}{r} 39 \\ 17 \\ \hline 273 \\ 39 \\ \hline 663 \end{array}$$