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7. STATE the lemma about the order of the element ab in terms of the order of a and the order of b.

Let a and b be elements of the group G. If $ab=ba$ and the order of a and the order of b are relatively prime, then the order of ab is equal to the order of a times the order of b

Pf let $r=$ order a, $s=$ order b, $t=$ order ab
$(ab)^{rs} \underset{\substack{\uparrow \\ G \text{ is abelian}}}{=} (a^r)^s(b^s)^r = id$ $\therefore t \mid rs$. On the other hand, $(ab)^t = id$ so $a^t = b^{-t}$ since G is abelian. Thus $a^t \in \langle a \rangle \cap \langle b \rangle$. Thus $o(a^t) \mid r$ and $o(a^t) \mid s$ but r and s are relatively prime so $o(a^t) = 1$ and $a^t = id$. Thus $r \mid t$. Also $b^t = id$ so $s \mid t$. But r and s are relatively prime so $rs \mid t$. We have positive integers t and rs with $t \mid rs$ and $rs \mid t$; thus $rs = t$.

8. Pick one of the statements from problems 5 through 7. Tell me which statement you have chosen. PROVE the statement.

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