

6. (6 points) TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If G is a group and $H = \{x^3 \mid x \in G\}$, then H is a subgroup of G .

False Take $G = D_3$. Then $H = \{e, \sigma, \sigma^2\}$ and this isn't a group because it isn't closed since $\sigma \cdot \sigma^2 = \rho \notin H$.

7. (5 points) Find the inverse of $[37]_{83}$ in $(\mathbb{Z}_{83}^\times, \times)$.

$$83 = 2 \cdot 37 + 9$$

$$37 = 4 \cdot 9 + 1$$

$$\text{So } 1 = 37 - 4 \cdot 9$$

$$1 = 37 - 4(83 - 2 \cdot 37)$$

$$1 = 9 \cdot 37 - 4 \cdot 83$$

$$\text{So } 9 \cdot 37 \equiv 1 \pmod{83}$$

and $[9]_{83}$ is the inverse of $[37]_{83}$ in \mathbb{Z}_{83}^\times