6. (6 points) TRUE or FALSE. (If true, PROVE it. If false, give a COUNTER EXAMPLE.) If $G$ is a group and $H = \{x^3 \mid x \in G\}$, then $H$ is a subgroup of $G$.

**False**

Take $G = D_3$. Then $H = \{1, \sigma, \sigma^2 \}$ and this isn't a group because it isn't closed since $\sigma \cdot \sigma^2 = \rho \notin H$.

7. (5 points) Find the inverse of $[37]_{83}$ in $(\mathbb{Z}_{83}^\times, \cdot)$.

$$37 = 2 \cdot 37 + 9$$

$$37 = 4 \cdot 9 + 1$$

So $1 = 37 - 4 \cdot 9$

$$1 = 37 - 4(83 - 2 \cdot 37)$$

$$1 = 9 \cdot 37 - 4 \cdot 83$$

So $9 \cdot 37 \equiv 1 \pmod{83}$

And $[9]_{83}$ is the inverse of $[37]_{83}$ in $\mathbb{Z}_{83}^\times$. 

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