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4. (5 points) Is $(\mathbb{Z}_{14}^{\times}, \times)$ a cyclic group? If so, exhibit a generator. If not, explain why not. $\mathbb{Z}_{14}^{\times}$ consists of $[1]_{14}, [3]_{14}, [5]_{14}, [9]_{14}, [11]_{14}, [13]_{14}$.

observe that $[3]_{14}^2 = [9]_{14}$, $[3]_{14}^3 = [13]_{14}$, $[3]_{14}^4 = [11]_{14}$, $[3]_{14}^5 = [5]_{14}$, $[3]_{14}^6 = [1]_{14}$

so $\mathbb{Z}_{14}^{\times}$ is cyclic with generator $[3]_{14}$.

5. (6 points) This problem has TWO parts. Let $(G, +)$ be an abelian group. Let $T_3(G) = \{g \in G \mid g + g + g = 0\}$.

(a) Prove $T_3(G)$ is a subgroup of G.

<u>0</u> $\in T_3(G)$ because $0+0+0=0$

<u>closure</u> If $g \in T_3(G)$ and $h \in T_3(G)$ then $(g+h)+(g+h)+(g+h) \overset{G \text{ is abelian}}{=} g+g+g+h+h+h$

$g, h \in T_3(G) \rightarrow$ $0+0=0$

<u>inverses</u> If $g \in T_3 G$, then $g+g+g=0$

so $g+g$ is the inverse of g. We have already seen that $T_3 G$ is closed so $g+g \in T_3 G$

(b) Compute $T_3(\mathbb{Z}_6, +)$.

$T_3(\mathbb{Z}_6) = \{[0]_6, [2]_6, [4]_6\}$ because

$[1]_6 + [1]_6 + [1]_6 = [3]_6 \neq [0]_6$

$[3]_6 + [3]_6 + [3]_6 = [3]_6 \neq [0]_6$

$[5]_6 + [5]_6 + [5]_6 = [3]_6 \neq [0]_6$